

Slope & $y = mx + b$

Algebra · Grade 8

★ Hard · 15 Questions

Solve each problem. Show your work in the space provided. Write your answer on the line.

1. Find the equation of the line through $(2, -3)$ and $(-4, 9)$.

Answer: _____

2. Solve the system: $y = 2x - 1$ and $y = -x + 5$.

Answer: _____

3. Two lines: $3x + 2y = 12$ and $6x + 4y = 24$. How many solutions does this system have?

Answer: _____

4. A company's profit P (in dollars) follows $P = 150x - 3,000$ where x is items sold. How many items must be sold to break even?

Answer: _____

5. Find the slope and y-intercept of $5x - 3y + 15 = 0$.

Answer: _____

6. A line L has slope $-3/4$. Line M is perpendicular to L and passes through $(0, 2)$. Write the equation of M.

Answer: _____

7. The point $(k, 2k-1)$ lies on $y = 3x - 7$. Find k.

Answer: _____

8. Find the equation of a line parallel to $2x - 5y = 10$ with the same y-intercept as $y = x + 4$.

Answer: _____

9. Solve the system: $2x + 3y = 12$ and $4x - y = 5$.

Answer: _____

10. A taxi charges \$2.50 base fare and \$1.80 per km. Another charges \$4.00 base and \$1.30 per km. At what distance do they cost the same?

Answer: _____

11. Line 1: passes through (1,3) and (3,7). Line 2: passes through (2,8) and (4,4). Do they intersect? If so, find the intersection.

Answer: _____

- 12.** A line passes through $(a, 0)$ and $(0, b)$ where $a+b=10$ and $ab=21$. Find the equation of the line.

Answer: _____

- 13.** The cost C of producing x units is $C=5x+200$. Revenue $R=12x$. Find the break-even quantity and profit at $x=50$.

Answer: _____

- 14.** Two lines are given by $y=mx+2$ and $y=3x+m$. For what value of m do they intersect at $x=1$?

Answer: _____

- 15.** A line has slope 3 and passes through the midpoint of $(-2,4)$ and $(6,-2)$. Find the equation.

Answer: _____

Answer Key

Slope & $y = mx + b$ — Hard

1. $y = -2x + 1$

$\text{slope} = (9 - (-3)) / (-4 - 2) = 12 / (-6) = -2$. $y + 3 = -2(x - 2)$. $y = -2x + 4 - 3 = -2x + 1$.

2. $x = 2, y = 3$

$2x - 1 = -x + 5$. $3x = 6$. $x = 2$. $y = 2(2) - 1 = 3$.

3. Infinitely many (same line)

Equation 2 = 2 $\times$ equation 1. Same line $\rightarrow$ infinite solutions.

4. 20 items

Set $P=0$: $150x = 3,000$. $x = 20$.

5. slope = $5/3$; y-intercept = 5

$3y = 5x + 15$. $y = (5/3)x + 5$.

6. $y = 4/3 x + 2$

Perpendicular slope = $4/3$. y-int = 2. $y = (4/3)x + 2$.

7. $k = 6$

Substitute: $2k - 1 = 3k - 7$. Subtract $2k$: $-1 = k - 7$. $k = 6$. Check: point $(6, 11)$; $y = 3(6) - 7 = 11$. $\checkmark$

8. $y = (2/5)x + 4$

Parallel slope: $2x - 5y = 10 \rightarrow y = (2/5)x - 2$. Slope = $2/5$. y-int from $y = x + 4$ is 4. $y = (2/5)x + 4$.

9. $x = 27/14 \approx 1.93, y = 19/7 \approx 2.71$

From eq2: $y = 4x - 5$. Sub: $2x + 3(4x - 5) = 12$. $14x = 27$. $x = 27/14$.

$y = 4(27/14) - 5 = 108/14 - 70/14 = 38/14 = 19/7$.

10. 3 km

$2.50 + 1.80d = 4.00 + 1.30d$. $0.50d = 1.50$. $d = 3$ km.

11. Yes — intersection at (2.75, 6.5)

Line 1: slope=2, $y=2x+1$. Line 2: slope=-2, $y=-2x+12$. $2x+1=-2x+12$. $4x=11$. $x=2.75$,
 $y=2(2.75)+1=6.5$.

12. One solution: a=3,b=7 or a=7,b=3. Line: $y=-7/3 x+7$ or $y=-3/7 x+3$.

$a+b=10$, $ab=21 \rightarrow a$ and b are roots of $t^2-10t+21=0$. $(t-3)(t-7)=0$. $t=3$ or 7 . Case 1: $x\text{-int}=3$, $y\text{-int}=7$.
slope=-7/3. $y=-(7/3)x+7$.

13. Break-even: $x \approx 28.6$ (29 units). Profit at 50: \$150.

Break-even: $12x=5x+200$. $7x=200$. $x \approx 28.6$. Profit= $R-C=12(50)-(5(50)+200)=600-450=\150 .

14. $m = 3 + m$... solve: at $x=1$: $m(1)+2=3(1)+m$. $m+2=3+m$. $2=3$. No solution — lines are parallel for all m .

At $x=1$: $m+2=3+m \rightarrow 2=3$. Contradiction. The lines never share a point at $x=1$ regardless of m ; they are parallel.

15. $y = 3x - 5$

Midpoint: $((-2+6)/2, (4+(-2))/2)=(2,1)$. $y-1=3(x-2)$. $y=3x-6+1=3x-5$. Check: $3(2)-5=1$. ✓
